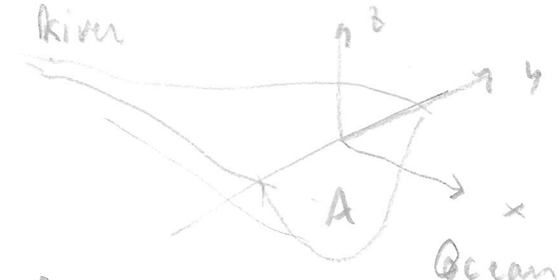
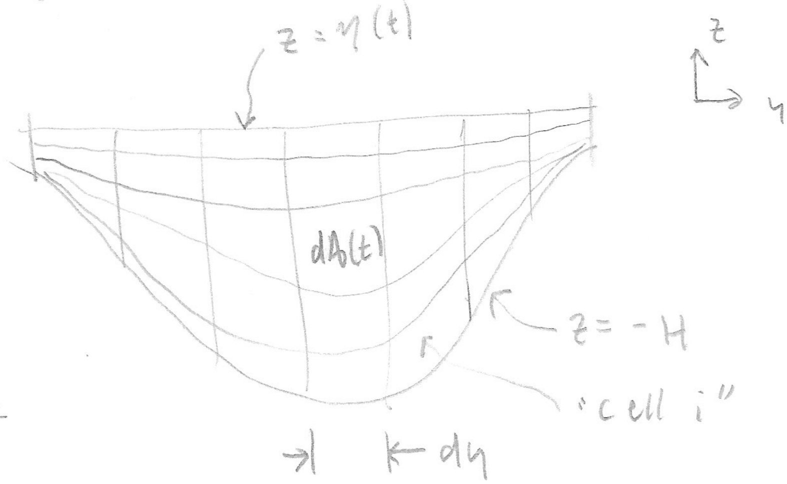
Estuarine Flux Decomposition

The salt flux through an estuary cross-section can be calculated in different ways, leading to different interpretations of the physics forcing exchange of ocean + estuary (mixed) water.

- Tidally averaged salt flux $\left\langle \int_A us\, dA \right\rangle$ is always the <u>goal</u>



- Because SSH η goes up and down, use σ-coordinates to calculate all fluxes:



$$\sigma \equiv \frac{z-\eta}{\eta+H}$$

$\sigma = -1$ at bottom, 0 at top

dA_i stretches + shrinks with tide height

Then, e.g., tidally-averaged velocity in a cell i is

$$\langle u_i \rangle \equiv \frac{\langle u_i \, dA_i \rangle}{\langle dA_i \rangle} \quad , \quad \text{centered on } \langle z_i \rangle \equiv \frac{\langle z_i \, dA_i \rangle}{\langle dA_i \rangle}$$

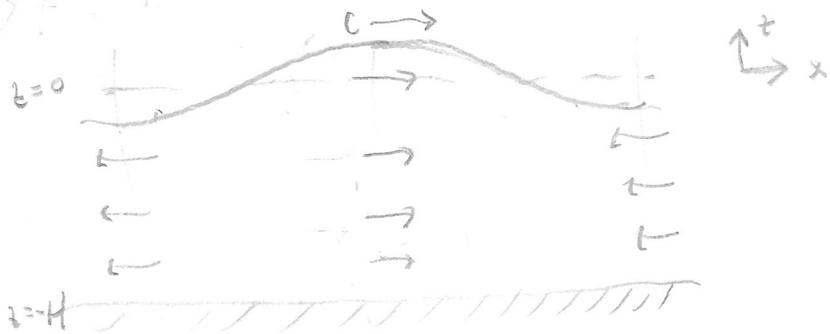
- Physically we would call this the "transport velocity".

- This distributes the Stokes Drift of progressive tidal waves throughout the water column.

in class Problem ☆ p.2.5

- E.g. for a progressive SW wave with

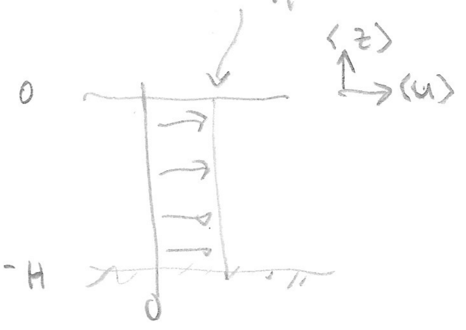
$u = u_0 \cos \omega t$, $h = H + \eta_0 \cos \omega t$ (at some x)



Averaging in σ coordinates:

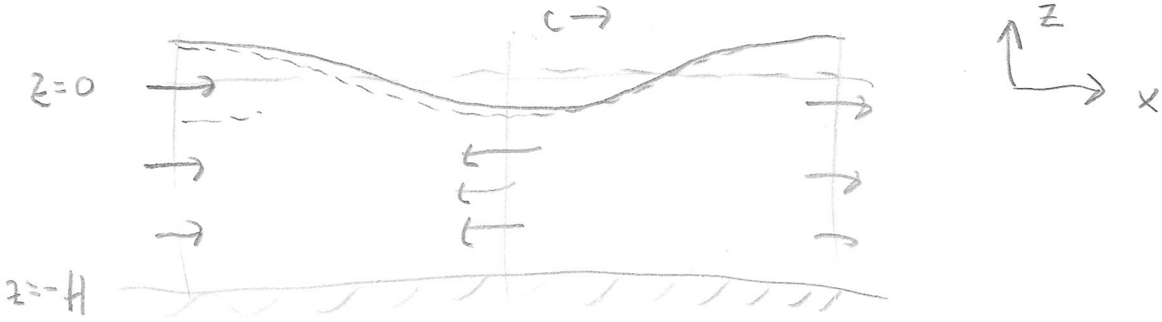
$$\Rightarrow \langle u \rangle = \frac{\langle u h \rangle}{\langle h \rangle} = \frac{\langle u_0 \cos \omega t \, H \rangle}{H} + \frac{\langle u_0 \eta_0 \cos^2 \omega t \rangle}{H}$$

$$\sim \langle u \rangle = \frac{1}{2} u_0 \frac{\eta_0}{H}$$ = the correct $\bar{u}$ to use for moving any tracer



(2.5)

☆ For a progressive SW wave



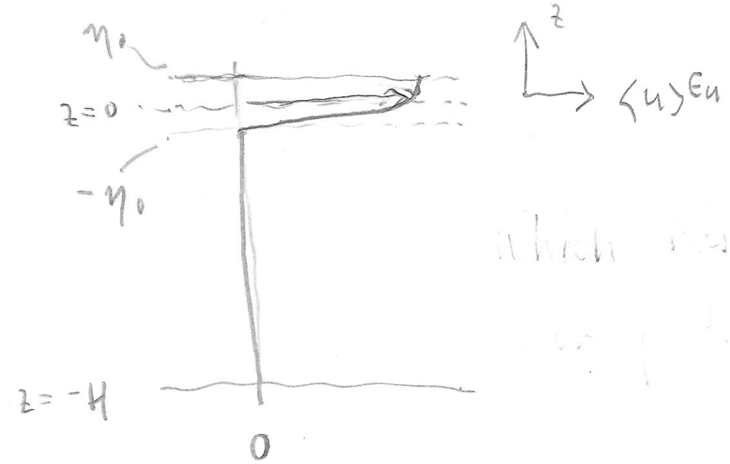
$$u = u_0 \cos(kx - \omega t)$$

$$\eta = \eta_0 \cos(kx - \omega t)$$

(1) what is the transport velocity

(2) what is the tidally averaged $u(z)$?

If instead we had used strict Eulerian averaging we would get



The vertical integral is the same, but the vertical profile of $\langle u\rangle^{Eu}$ has no relation to how tracers move.

Returning to salt flux

$$F_s = \left\langle \iint_A u\, s\, dA \right\rangle$$

and we can decompose variables as

$$u = u_0 + u_1 + u_2$$

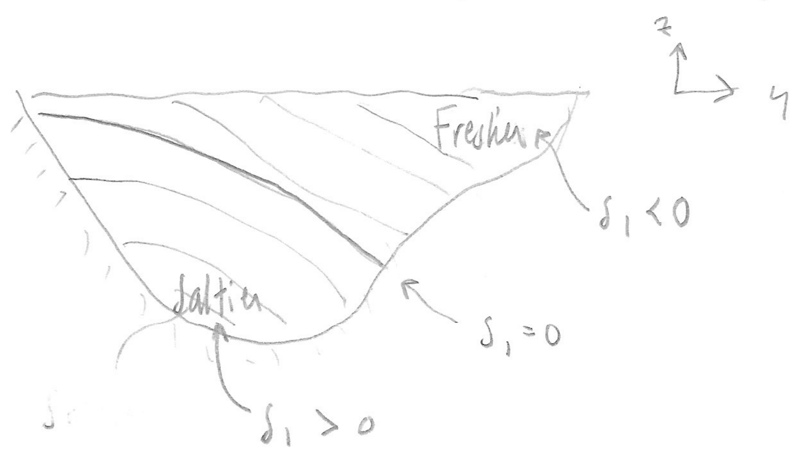
$$s = s_0 + s_1 + s_2$$

s_2 ← section-varying & tidally-varying

s_1 ← section-varying and tidally averaged

s_0 ← section-averaged and tidally-averaged (same as $\bar{u}$ & $\bar{s}$)

e.g.



similar to $s'(z)$ from previous lectures but we are allowing more spatial structure $s_1(y, z)$

e.g. to allow for tilt due to Coriolis.

Then

$$dA_0 = \langle dA \rangle$$

$$F_s = \underbrace{u_0 s_0}_{\text{like } \bar{u}\bar{s}} + \underbrace{\int u_1 s_1 \, dA_0}_{\text{like } \overline{u's'}} + \underbrace{\left\langle \int u_2 s_2 \, dA \right\rangle}$$

like $\bar{u}\,\bar{s}$ = "River"

like $\overline{u's'}$ = "Exchange Flow"

new term to account for salt transport that is mostly due to tidal correlation of u + s

e.g. when isohalines slosh back + forth

And typically the tidal term is parameterized as Fickean diffusion:

$$\left\langle \int u_2 s_2 \, dA \right\rangle = -K_H \frac{\partial s_0}{\partial x}$$

- Tidal term brings salt into the estuary (like the exchange flow)
- $K_H \gg K$ e.g. $K_H \propto 0.05\, U_T L_T \sim 10 - 100\ m^2/s$

⇒ - Result for physics: need theory for u_1 + s_1 (like u' + s') and now also for K_H.